P859

14.1 Introduction to Rational Expressions

Basic Concepts • Simplifying Rational Expressions • Applications

A LOOK INTO MATH ▸

Have you ever been moving smoothly in traffic, only to come to a sudden halt? Mathematics shows that in certain conditions, if the number of cars on a road increases even slightly, then the movement of traffic can slow dramatically. To understand why this occurs, we will consider how *rational expressions* can be used to model traffic flow. (See Example 6.)

NEW VOCABULARY

- ☐ Rational expression
- ☐ Lowest terms
- ☐ Vertical asymptote
- ☐ Probability

Basic Concepts

Recall that a *rational number* is any number that can be expressed as a ratio of two integers $\frac{p}{q}$, where $q \neq 0$. In this chapter, we discuss *rational expressions*, which can be written as the ratio of two polynomials. Because examples of polynomials include

$$3, \quad 2x, \quad x^2 + 4, \quad \text{and} \quad x^3 - 1,$$

it follows that examples of rational expressions include

$$\frac{3}{2x}, \quad \frac{2x}{x^2 + 4}, \quad \frac{x^2 + 4}{3}, \quad \text{and} \quad \frac{x^3 - 1}{x^2 + 4}.$$

RATIONAL EXPRESSION

A **rational expression** can be written as $\frac{P}{Q}$, where P and Q are polynomials. A rational expression is defined whenever $Q \neq 0$.

We can evaluate polynomials for different values of a variable. For example, for $x = 2$ the polynomial $x^2 - 3x + 1$ evaluates to

$$(2)^2 - 3(2) + 1 = -1.$$

Rational expressions can be evaluated similarly.

EXAMPLE 1 **Evaluating rational expressions**

If possible, evaluate each expression for the given value of the variable.

(a) $\frac{1}{x + 1}$ $\quad x = 2$ **(b)** $\frac{y^2}{2y - 1}$ $\quad y = -4$

(c) $\frac{5z + 8}{z^2 - 2z + 1}$ $\quad z = 1$ **(d)** $\frac{2 - x}{x - 2}$ $\quad x = -3$

TEACHING EXAMPLE 1

Repeat Example 1 for each of the following.

(a) $\frac{3}{x - 1}$; $x = 3$

(b) $\frac{t^2 - 1}{\ }$; $t = -2$

a) $\frac{1}{x+1}$ $\quad x = 2$

b) $\frac{5z + 8}{z^2 - 2z + 1}$ $\quad z = 1$

READING CHECK

- When is a rational expression undefined?

UNDEFINED EXPRESSIONS Division by 0 is undefined. As a result, rational expressions are different from polynomials because they are *undefined* whenever their *denominators are 0*. For example, the expression in Example 1(c) is undefined when $z = 1$.

EXAMPLE 2

Determining when a rational expression is undefined

TEACHING EXAMPLE 2

Repeat Example 2 for each of the following expressions.

(a) 16 (b) 5y

Find all values of the variable for which each expression is undefined.

(a) $\frac{1}{x}$ **(b)** $\frac{4t}{t - 3}$ **(c)** $\frac{1 - 6r}{r^2 - 4}$ **(d)** $\frac{4}{x^2 + 1}$

a) $\frac{1}{x}$ b)

STUDY TIP

You may want to review your notes on fractions. Many of the mathematical concepts that apply to fractions also apply to rational expressions. Fractions were discussed in Chapter 4.

Simplifying Rational Expressions

In Chapter 4, we used the *basic principle of fractions*,

$$\frac{a \cdot c}{b \cdot c} = \frac{a}{b}.$$

For example, this basic principle allows us to simplify the fraction $\frac{8}{12}$ as

$$\frac{8}{12} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{2}{3}.$$

EXAMPLE 3

Simplifying fractions

TEACHING EXAMPLE 3

Repeat Example 3 for each of the following fractions.

(a) $\frac{18}{24}$ (b) $-\frac{32}{40}$

ANS. (a) $\frac{3}{4}$ (b) $-\frac{4}{5}$

Simplify each fraction by applying the basic principle of fractions.

(a) $\frac{5}{10}$ **(b)** $-\frac{36}{48}$

Solution

(a) $\frac{5}{10} = \frac{1 \cdot 5}{2 \cdot 5} = \frac{1}{2}$ **(b)** $-\frac{36}{48} = -\frac{3 \cdot 12}{4 \cdot 12} = -\frac{3}{4}$

Now Try Exercises 39, 43

We can also apply this basic principle to rational expressions. For example,

$$\frac{x(x - 1)}{4(x - 1)} = \frac{x}{4},$$

provided that $x \neq 1$.

NOTE: The simplification at the bottom of the previous page is not valid when $x = 1$ because the expression is undefined for this x-value. When simplifying a rational expression, *we assume that values of the variable that make the rational expression undefined are excluded*, unless stated otherwise.

BASIC PRINCIPLE OF RATIONAL EXPRESSIONS

The following property can be used to simplify rational expressions, where P, Q, and R are polynomials.

$$\frac{P \cdot R}{Q \cdot R} = \frac{P}{Q} \quad Q \text{ and } R \text{ are nonzero.}$$

NOTE: $\frac{P \cdot R}{Q \cdot R} = \frac{P}{Q} \cdot \frac{R}{R} = \frac{P}{Q} \cdot 1 = \frac{P}{Q}$, provided that $Q \neq 0$ and $R \neq 0$.

Like fractions, rational expressions can be written in *lowest terms*. For example, the rational expression $\frac{x^2 - 1}{x^2 + 2x + 1}$ can be written in lowest terms by factoring the numerator and the denominator and then applying the basic principle of rational expressions.

READING CHECK

- How do you know when a rational expression is written in lowest terms?

$$\frac{x^2 - 1}{x^2 + 2x + 1} = \frac{(x - 1)(x + 1)}{(x + 1)(x + 1)} \quad \text{Factor the numerator and the denominator.}$$

$$= \frac{x - 1}{x + 1} \quad \text{Apply } \frac{PR}{QR} = \frac{P}{Q} \text{ with } R = x + 1.$$

Because the basic principle of rational expressions cannot be applied further to $\frac{x - 1}{x + 1}$, we say that this expression is written in **lowest terms**.

EXAMPLE 4 Simplifying rational expressions

TEACHING EXAMPLE 4
Simplify each expression.
(a) $\frac{12x^2}{4x^3}$ (b) $\frac{4y - 16}{5y - 20}$
(c) $\frac{(w - 3)(w + 6)}{(w + 6)(w + 1)}$
(d) $\frac{x^2 - 3x + 2}{x^2 - 4}$
ANS. (a) $\frac{3}{x}$ (b) $\frac{4}{5}$ (c) $\frac{w - 3}{w + 1}$
(d) $\frac{x - 1}{x + 2}$

Simplify each expression.

(a) $\frac{8y}{4y^2}$ **(b)** $\frac{2x + 6}{3x + 9}$ **(c)** $\frac{(z + 1)(z - 5)}{(z - 5)(z + 3)}$ **(d)** $\frac{x^2 - 9}{2x^2 + 7x + 3}$

Solution

(a) Factor out the greatest common factor, $4y$, in the numerator and the denominator.

$$\frac{8y}{4y^2} = \frac{2 \cdot 4y}{y \cdot 4y} = \frac{2}{y} \quad \text{Apply } \frac{PR}{QR} = \frac{P}{Q} \text{ with } R = 4y.$$

(b) Start by factoring the numerator and denominator.

$$\frac{2x + 6}{3x + 9} = \frac{2(x + 3)}{3(x + 3)} = \frac{2}{3} \quad \text{Apply } \frac{PR}{QR} = \frac{P}{Q} \text{ with } R = x + 3.$$

(c) The commutative property allows us to write $\frac{PR}{QR}$ as $\frac{PR}{RQ}$.

$$\frac{(z + 1)(z - 5)}{(z - 5)(z + 3)} = \frac{z + 1}{z + 3} \quad \text{Apply } \frac{PR}{RQ} = \frac{P}{Q} \text{ with } R = z - 5.$$

(d) Start by factoring the numerator and the denominator.

$$\frac{x^2 - 9}{2x^2 + 7x + 3} = \frac{(x - 3)(x + 3)}{(2x + 1)(x + 3)} = \frac{x - 3}{2x + 1} \quad \text{Apply } \frac{PR}{QR} = \frac{P}{Q} \text{ with } R = x + 3.$$

Now Try Exercises 51, 55, 61, 79

MAKING CONNECTIONS

Expressions and Equations

Expressions and equations are different concepts. An expression does not contain an equals sign, whereas an equation is a statement that two expressions are equal and *always* contains an equals sign. For example,

$$\frac{x}{x+4} \quad \text{and} \quad \frac{2}{x}$$

are two rational *expressions*, and

$$\frac{x}{x+4} = \frac{2}{x} \quad \text{An equation must contain an equals sign.}$$

is a rational *equation*. In this section we evaluate and simplify rational expressions. Later, we will *solve* rational equations by finding x-values that make the equations true.

READING CHECK

- How do rational expressions and rational equations differ?

A negative sign can be placed in a fraction in a number of ways. For example,

$$-\frac{5}{7} = \frac{-5}{7} = \frac{5}{-7}$$

illustrates three fractions that are equal. This property can also be applied to rational expressions, as demonstrated in the next example.

EXAMPLE 5 **Distributing a negative sign**

Simplify each expression.

(a) $\frac{-x-6}{2x+12}$ **(b)** $\frac{10-z}{z-10}$ **(c)** $-\frac{5-x}{x-5}$

TEACHING EXAMPLE 5
Simplify each expression.
(a) $\frac{-y-3}{4y+12}$ (b) $\frac{x-7}{7-x}$
(c) $-\frac{18-t}{t-18}$
ANS. (a) $-\frac{1}{4}$ (b) -1 (c) 1

Solution

(a) Factor -1 out of the numerator and 2 out of the denominator.

$$\frac{-x-6}{2x+12} = \frac{-1(x+6)}{2(x+6)} = -\frac{1}{2}$$

(b) Factor -1 out of the numerator.

$$\frac{10-z}{z-10} = \frac{-1(-10+z)}{z-10} = \frac{-1(z-10)}{z-10} = -1$$

(c) Rewrite the expression with the negative sign in the numerator and then apply the distributive property. Be sure to include parentheses around the numerator.

$$-\frac{5-x}{x-5} = \frac{-(5-x)}{x-5} = \frac{-5+x}{x-5} = \frac{x-5}{x-5} = 1$$

The same answer can be obtained by distributing the negative sign in the denominator.

$$-\frac{5-x}{x-5} = \frac{5-x}{-(x-5)} = \frac{5-x}{-x+5} = \frac{5-x}{5-x} = 1$$

Now Try Exercises 63, 67, 71

NOTE: The result for Example 5(b) becomes more obvious if we substitute a number for z. For example, if we let $z = 6$, then

$$\frac{10 - z}{z - 10} = \frac{10 - 6}{6 - 10} = \frac{4}{-4} = -1.$$

TEACHING TIP

The order of a subtraction can be changed by factoring out -1.

MAKING CONNECTIONS

Negative Signs and Rational Expressions

In general, $(b - a)$ equals $-1(a - b)$. Thus if $a \neq b$, then $\frac{b - a}{a - b} = -1$.

Applications

▶ **REAL-WORLD CONNECTION** The next example is based on the discussion in A Look Into Math for this section and illustrates modeling traffic flow with a rational expression.

EXAMPLE 6 **Modeling traffic flow**

TEACHING EXAMPLE 6
For a similar example, do Exercise 100.

Suppose that 10 cars per minute can pass through a construction zone. If traffic arrives *randomly* at an average rate of x cars per minute, the average time T in minutes spent waiting in line and passing through the construction zone is given by

$$T = \frac{1}{10 - x},$$

where $x < 10$. (*Source:* N. Garber and L. Hoel, *Traffic and Highway Engineering.*)

(a) Complete Table 14.1 by finding T for each value of x.

TABLE 14.1 Waiting in Traffic

x (cars/minute)	5	7	9	9.5	9.9	9.99
T (minutes)						

(b) Interpret the results.

Solution

(a) When $x = 5$ cars per minute, then $T = \frac{1}{10 - 5} = \frac{1}{5}$ minute. Other values are found similarly and are shown in Table 14.2.

TABLE 14.2 Waiting in Traffic

x (cars/minute)	5	7	9	9.5	9.9	9.99
T (minutes)	$\frac{1}{5}$	$\frac{1}{3}$	1	2	10	100

(b) As the average traffic rate increases from 9 cars per minute to 9.9 cars per minute, the time needed to pass through the construction zone increases from 1 minute to 10 minutes. As x nears 10 cars per minute, a small increase in x increases the waiting time dramatically.

Now Try Exercise 99

This nonlinear effect for traffic congestion in Example 6 is shown in Figure 14.1, where points from Table 14.2 have been plotted and a curve passing through them has been sketched. A vertical dashed line was also sketched at $x = 10$. This dashed line is called a

vertical asymptote and indicates that the rational expression is undefined at this value of x. Near the left side of the vertical asymptote, the waiting time T increases dramatically for small increases in x. The graph of T does not intersect or cross this vertical asymptote.

TEACHING TIP

Point out that a slight increase in the average traffic rate x can sometimes cause a dramatic increase in the waiting time on a congested road.

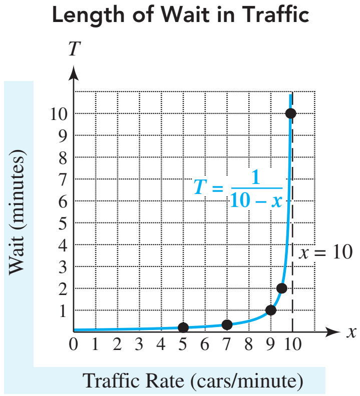
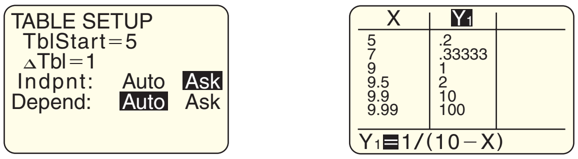


Figure 14.1

TECHNOLOGY NOTE

Making Tables

Table 14.2 can also be created with a graphing calculator by using the Ask feature, as illustrated in the following displays.



CALCULATOR HELP

To make a table of values, see Appendix A (pages AP-2 and AP-3).

▶ **REAL-WORLD CONNECTION** When a new species of animal is introduced into an area that it did not previously inhabit, its population may grow quickly at first, and then level off over time. Rational expressions can be used to model such situations, as demonstrated in the next example.

EXAMPLE 7 **Modeling a fish population**

TEACHING EXAMPLE 7

For a similar example, do Exercise 102.

Suppose that a small fish species is introduced into a pond that had not previously held this type of fish, and that its population P in thousands is modeled by

$$P = \frac{3x + 1}{x + 4},$$

where $x \geq 0$ represents time in months.

(a) Complete Table 14.3 by finding P for each value of x. Round to 3 decimal places.

TABLE 14.3 **Fish Population**

x (months)	0	6	12	36	72
P (thousands)					

(b) How many fish were initially introduced into the pond?

(c) Interpret the results shown in your completed table.

Solution

(a) When $x = 0$, the population is $P = \frac{3(0) + 1}{0 + 4} = \frac{1}{4} = 0.25$ thousand fish. The other values are found similarly and are shown in Table 14.4.

TABLE 14.4 Fish Population

x (months)	0	6	12	36	72
P (thousands)	0.25	1.9	2.313	2.725	2.855

(b) Table 14.4 shows that initially (when $x = 0$) there were 0.25 thousand, or 250 fish.

(c) The fish population increased quickly at first but then leveled off. This population growth is shown graphically in Figure 14.2, where the population appears to be leveling off at 3 thousand fish.

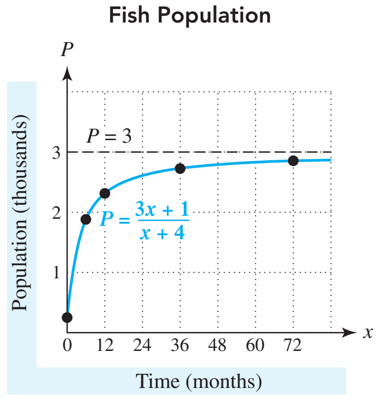


Figure 14.2

Now Try Exercise 101

TEACHING EXAMPLE 8
Repeat Example 8 if two balls in the container have the winning number.
ANS. (a) $\frac{2}{n}$ (b) $\frac{1}{50}$; $\frac{1}{500}$; $\frac{1}{5000}$
(c) The probability decreases.

AN APPLICATION INVOLVING PROBABILITY (OPTIONAL) If 10 marbles, one blue and nine red, are placed in a jar, then the *probability*, or *likelihood,* of picking the blue marble at random is 1 *chance* in 10, or $\frac{1}{10}$. The probability of drawing a red marble at random is 9 chances in 10, or $\frac{9}{10}$. **Probability** is a real number from 0 to 1. A probability of 0, or 0%, indicates that an event is impossible, whereas a probability of 1, or 100%, indicates that an event is certain. Rational expressions are often used to describe probability.

EXAMPLE 8 Calculating probability

Suppose that n balls, numbered 1 to n, are placed in a container and only one ball has the winning number.

(a) What is the probability of drawing the winning ball at random?

(b) Calculate this probability for $n = 100$, 1000, and 10,000.

(c) What happens to the probability of drawing the winning ball as the number of balls increases?

Solution

(a) There is 1 chance in n of drawing the winning ball, so the probability is $\frac{1}{n}$.

(b) For $n = 100$, 1000, and 10,000, the probabilities are $\frac{1}{100}$, $\frac{1}{1000}$, and $\frac{1}{10{,}000}$.

(c) As the number of balls increases, the probability of picking the winning ball decreases.

Now Try Exercise 105

14.1 Putting It All Together

CONCEPT	EXPLANATION	EXAMPLES
Rational Expression	An expression of the form $\frac{P}{Q}$, where P and Q are polynomials with $Q \neq 0$	$\frac{1}{x}$, $\frac{x-3}{2x^2-1}$, $\frac{2x+9}{5x}$, and $\frac{x^2+3x-5}{1}$
Undefined Rational Expressions	A rational expression is undefined for any value of the variable that makes the *denominator* equal to 0.	$\frac{1}{x-3}$ is undefined when $x = 3$. $\frac{5y}{y^2-1}$ is undefined when $y = 1$ or when $y = -1$.
Basic Principle of Rational Expressions	Factor the numerator and the denominator completely. Then apply $\frac{P \cdot R}{Q \cdot R} = \frac{P}{Q}$.	$\frac{4xy^2}{6xy^3} = \frac{2(2xy^2)}{3y(2xy^2)} = \frac{2}{3y}$ $\frac{4x(x-4)}{(x+1)(x-4)} = \frac{4x}{x+1}$

14.1 Exercises

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CONCEPTS AND VOCABULARY

1. A rational expression can be written as _____, where P and Q are _____ with $Q \neq 0$. $\frac{P}{Q}$; polynomials
2. Is $\frac{x}{2x^2+1}$ a rational expression? Why or why not? Yes; both x and $2x^2+1$ are polynomials.
3. A rational expression is undefined whenever the _____ is equal to 0. denominator
4. The rational expression $\frac{1}{x-a}$ is undefined whenever $x =$ __a__.
5. The basic principle of fractions states that a fraction can be simplified by using $\frac{a \cdot c}{b \cdot c} =$ __$\frac{a}{b}$__.
6. The basic principle of rational expressions can be used to simplify _____ expressions. rational

EVALUATING RATIONAL EXPRESSIONS

Exercises 7–20: If possible, evaluate the expression for the given value of the variable.

7. $\frac{3}{x}$; $x = -7$ $-\frac{3}{7}$
8. $\frac{3}{x+3}$; $x = 0$ 1
9. $-\frac{x}{x-5}$; $x = -4$ $-\frac{4}{9}$
10. $-\frac{4x}{5x+1}$; $x = 1$ $-\frac{2}{3}$
11. $\frac{y+1}{y^2}$; $y = -2$ $-\frac{1}{4}$
12. $\frac{3y-1}{y^2+1}$; $y = -1$ -2
13. $\frac{7z}{z^2-4}$; $z = -2$ Undefined
14. $\frac{5}{z^2-3z+2}$; $z = -1$ $\frac{5}{6}$
15. $\frac{5}{3t+6}$; $t = -2$ Undefined
16. $\frac{4t}{2t+5}$; $t = -\frac{5}{2}$ Undefined
17. $\frac{4-x}{x-4}$; $x = -2$ -1
18. $\frac{x-7}{7-x}$; $x = 4$ -1
19. $-\frac{6-x}{x-6}$; $x = 0$ 1
20. $\frac{8-2x}{2x-8}$; $x = -5$ -1

Exercises 21–24: Complete the table for the given expression. If a value is undefined, place a dash in the table.

21.

x	-2	-1	0	1	2
$\frac{x}{x+1}$	2	—	0	$\frac{1}{2}$	$\frac{2}{3}$

22.

x	-2	-1	0	1	2
$\frac{2x}{3x-1}$	$\frac{4}{7}$	$\frac{1}{2}$	0	1	$\frac{4}{5}$

23.

x	-2	-1	0	1	2
$\dfrac{3x}{2x^2 + 1}$	$-\frac{2}{3}$	-1	0	1	$\frac{2}{3}$

24.

x	-2	-1	0	1	2
$\dfrac{2x - 1}{x^2 - 1}$	$-\frac{5}{3}$	—	1	—	1

Exercises 25–38: Find any values of the variable that make the expression undefined.

25. $-\dfrac{8}{x}$ 0

26. $\dfrac{7}{x + 1}$ -1

27. $\dfrac{4}{z - 3}$ 3

28. $\dfrac{7 - z}{z - 7}$ 7

29. $\dfrac{4y}{5y + 4}$ $-\frac{4}{5}$

30. $\dfrac{3 + y}{3y - 7}$ $\frac{7}{3}$

31. $\dfrac{5t + 2}{t^2 + 1}$ None

32. $\dfrac{8t}{t^2 + 25}$ None

33. $\dfrac{8x}{x^2 - 25}$ $-5, 5$

34. $\dfrac{x + 4}{x^2 - 36}$ $-6, 6$

35. $\dfrac{x^2 + 3x + 2}{x^2 + 5x + 6}$ $-3, -2$

36. $\dfrac{2x - 1}{x^2 - 7x + 10}$ $2, 5$

37. $\dfrac{8z^2 + z + 1}{2z^2 - 7z + 5}$ $1, \frac{5}{2}$

38. $\dfrac{4n^2 + 17n - 15}{3n^2 - 8n + 4}$ $\frac{2}{3}, 2$

SIMPLIFYING RATIONAL EXPRESSIONS

Exercises 39–46: Simplify the fraction to lowest terms.

39. $\frac{12}{18}$ $\frac{2}{3}$

40. $\frac{24}{32}$ $\frac{3}{4}$

41. $\frac{24}{48}$ $\frac{1}{2}$

42. $-\frac{22}{33}$ $-\frac{2}{3}$

43. $-\frac{6}{15}$ $-\frac{2}{5}$

44. $\frac{8}{22}$ $\frac{4}{11}$

45. $-\frac{25}{75}$ $-\frac{1}{3}$

46. $-\frac{36}{42}$ $-\frac{6}{7}$

Exercises 47–50: First simplify the fraction in part (a), then simplify the rational expression in part (b).

47. (a) $\dfrac{8}{16}$ $\frac{1}{2}$ **(b)** $\dfrac{x + 2}{2x + 4}$ $\frac{1}{2}$

48. (a) $\dfrac{6}{9}$ $\frac{2}{3}$ **(b)** $\dfrac{4x + 12}{6x + 18}$ $\frac{2}{3}$

49. (a) $\dfrac{7 - 3}{3 - 7}$ -1 **(b)** $\dfrac{7 - x}{x - 7}$ -1

50. (a) $-\dfrac{8 - 5}{5 - 8}$ 1 **(b)** $-\dfrac{x - 5}{5 - x}$ 1

Exercises 51–88: Simplify the expression.

51. $\dfrac{5x^4}{10x^6}$ $\frac{1}{2x^2}$

52. $\dfrac{6y^2}{9y}$ $\frac{2y}{3}$

53. $\dfrac{8xy^3}{6x^2y^2}$ $\frac{4y}{3x}$

54. $\dfrac{36x^2y^5}{6x^5y}$ $\frac{6y^4}{x^3}$

55. $\dfrac{x + 4}{2x + 8}$ $\frac{1}{2}$

56. $\dfrac{5x - 10}{x - 2}$ 5

57. $\dfrac{3z - 9}{5z - 15}$ $\frac{3}{5}$

58. $\dfrac{4z + 8}{10 + 5z}$ $\frac{4}{5}$

59. $\dfrac{(x + 1)(x - 1)}{(x + 6)(x - 1)}$ $\frac{x + 1}{x + 6}$

60. $\dfrac{(2x + 1)(x + 9)}{(4x - 3)(x + 9)}$ $\frac{2x + 1}{4x - 3}$

61. $\dfrac{(5y + 3)(2y - 1)}{(2y - 1)(y + 2)}$ $\frac{5y + 3}{y + 2}$

62. $\dfrac{(4y - 1)(5y + 7)}{(5y + 7)(1 - 4y)}$ -1

63. $\dfrac{x - 7}{7 - x}$ -1

64. $\dfrac{5 - x}{x - 5}$ -1

65. $\dfrac{a - b}{b - a}$ -1

66. $\dfrac{2t - 3r}{3r - 2t}$ -1

67. $\dfrac{-6 - x}{18 + 3x}$ $-\frac{1}{3}$

68. $\dfrac{-2x - 6}{x + 3}$ -2

69. $\dfrac{x + 1}{-2x - 2}$ $-\frac{1}{2}$

70. $\dfrac{3x + 21}{-7 - x}$ -3

71. $-\dfrac{9 - x}{x - 9}$ 1

72. $-\dfrac{4 - 2x}{x - 2}$ 2

73. $\dfrac{(3x + 5)(x - 1)}{(3x - 5)(1 - x)}$ $-\frac{3x + 5}{3x - 5}$

74. $\dfrac{(2 - x)(x - 2)}{(x - 2)(2 - x)}$ 1

75. $\dfrac{n^2 - n}{n^2 - 5n}$ $\frac{n - 1}{n - 5}$

76. $\dfrac{3n^2 - 4n}{n^2 + 4n}$ $\frac{3n - 4}{n + 4}$

77. $\dfrac{x^2 - 3x}{6x - 18}$ $\frac{x}{6}$

78. $\dfrac{4x^2 + 16x}{5x^2 + 20x}$ $\frac{4}{5}$

79. $\dfrac{z^2 - 3z + 2}{z^2 - 4z + 3}$ $\frac{z - 2}{z - 3}$

80. $\dfrac{z^2 - 3z - 10}{z^2 - 2z - 8}$ $\frac{z - 5}{z - 4}$

81. $\dfrac{2x^2 + 7x - 4}{6x^2 + x - 2}$ $\frac{x + 4}{3x + 2}$

82. $\dfrac{5x^2 + 3x - 2}{5x^2 + 13x - 6}$ $\frac{x + 1}{x + 3}$

83. $\dfrac{x - 3}{3x^2 - 11x + 6}$ $\frac{1}{3x - 2}$

84. $\dfrac{2x - 1}{4x^2 + 6x - 4}$ $\frac{1}{2x + 4}$

85. $-\dfrac{a - 9}{9 - a}$ 1

86. $-\dfrac{-b - 6}{b + 6}$ 1

87. $\dfrac{-2x - 1}{4x + 2}$ $-\frac{1}{2}$

88. $\dfrac{4x + 3}{-8x - 6}$ $-\frac{1}{2}$

Exercises 89 and 90: **Thinking Generally** *Complete the statement involving a rational expression.*

89. The expression $\frac{x-a}{a-x}$ simplifies to $\underline{-1}$.

90. The expression $-\frac{x-a}{x+a}$ simplifies to $\frac{?}{x+a}$. $a - x$

Exercises 91–98: Refer to Making Connections on page 861.
(a) Decide whether you are given an expression or an equation.
(b) If you are given an expression, simplify it. If you are given an equation, solve it.

91. $x + 1 = 7$
(a) Equation (b) 6

92. $x^2 - 4 = 0$
(a) Equation (b) $-2, 2$

93. $\frac{x}{x(x+1)}$
(a) Expression (b) $\frac{1}{x+1}$

94. $\frac{x-2}{(x-2)(x-8)}$
(a) Expression (b) $\frac{1}{x-8}$

95. $\frac{x^2-4}{x+2}$
(a) Expression (b) $x - 2$

96. $\frac{x-4}{8-2x}$
(a) Expression (b) $-\frac{1}{2}$

97. $\frac{x}{2(1+3)} = 1$
(a) Equation (b) 8

98. $\frac{x}{2} + 2 = \frac{8}{2}$
(a) Equation (b) 4

APPLICATIONS

99. (a) $\frac{1}{2}$; when traffic arrives at an average rate of 3 vehicles/min, the average wait is $\frac{1}{2}$ min.

99. ***Modeling Traffic Flow*** (Refer to Example 6.) Five vehicles per minute can pass through a construction zone. If the traffic arrives randomly at an average rate of x vehicles per minute, the average time T in minutes spent waiting in line and passing through the construction zone is given by $T = \frac{1}{5-x}$ for $x < 5$. (*Source:* N. Garber.)

(a) Evaluate T for $x = 3$ and interpret the result.

(b) Complete the table and interpret the results.
As x nears 5 vehicles/min, a small increase in x increases the wait dramatically.

x	2	4	4.5	4.9	4.99
T	$\frac{1}{3}$	1	2	10	100

100. ***Standing in Line*** A worker at a poolside store can serve 20 customers per hour. If children arrive randomly at an average rate of x per hour, then the average number of children N waiting in line is given by $N = \frac{x^2}{400 - 20x}$ for $x < 20$. (*Source:* N. Garber.)

(a) Complete the table.

x	5	10	18	19	20
N	$0.08\overline{3}$	0.5	8.1	18.05	—

(b) Compare the number of children waiting in line if the average rate increases from 18 to 19 children per hour. It increases by almost 10 children.

101. ***Frog Population*** Suppose that a frog species is introduced into a wetland area and its population in hundreds is modeled by

$$P = \frac{7x+3}{x+6},$$

where $x \geq 0$ is time in months.

(a) Complete the table by finding P for each given value of x. Round to 2 decimal places.

x (months)	0	12	36	72
P (hundreds)	0.5	4.83	6.07	6.5

(b) What was the initial frog population? 50

(c) Interpret the results in your completed table.
The population increased quickly at first, but then leveled off.

102. ***Insect Population*** Suppose that an insect population in thousands per acre is modeled by

$$P = \frac{5x+2}{x+1},$$

where $x \geq 0$ is time in months.

(a) Complete the table by finding P for each given value of x. Round to 3 decimal places.

x (months)	0	12	36	60
P (thousands)	2	4.769	4.919	4.951

(b) What was the initial insect population? 2000

(c) Interpret the results in your completed table.
The population increased quickly at first, but then leveled off.

103. ***Probability*** Suppose that a coin is flipped. What is the probability that a head appears? $\frac{1}{2}$

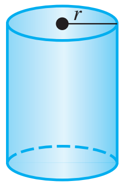
104. ***Probability*** A die shows the numbers 1, 2, 3, 4, 5, and 6. If each number has an equal chance of appearing on any given roll, what is the probability that a 2 or 4 appears? $\frac{2}{6}$, or $\frac{1}{3}$

105. ***Probability*** (Refer to Example 8.) Suppose that there are n balls in a container and that three balls have a winning number. If a ball is drawn randomly, do each of the following.

(a) Write a rational expression that gives the probability of drawing a winning ball. $\frac{3}{n}$

(b) Write a rational expression that gives the probability of not drawing a winning ball. Evaluate your expression for $n = 100$ and interpret the result.
$\frac{n-3}{n}$; $\frac{97}{100}$; there is a 97% chance that a winning ball will not be drawn.

106. ***Surface Area of a Cylinder*** If a cylindrical container has a volume of π cubic feet, then its surface area S in square feet (excluding the top and bottom) is given by $S = \frac{2\pi}{r}$, where r is the radius of the cylinder.



(a) Calculate S when $r = \frac{1}{2}$ foot. $4\pi \approx 12.6 \text{ ft}^2$

(b) What happens to this surface area when r becomes large? Sketch this situation.* It becomes very small.

(c) What happens to the surface area when r becomes small (nearly 0)? Sketch this situation.* It becomes very large.

107. ***Distance and Time*** A car is traveling at 60 miles per hour. (a) 6 hr

(a) How long does it take the car to travel 360 miles?

(b) Write a rational expression that gives the time that it takes the car to travel M miles. $\frac{M}{60}$

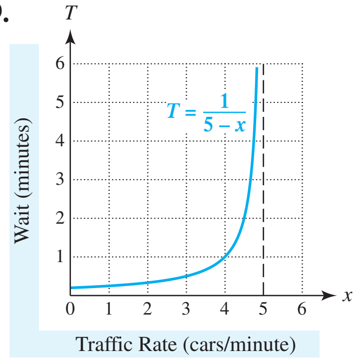
108. ***Distance and Time*** A bicyclist rides uphill at 10 miles per hour for 5 miles and then rides downhill at 20 miles per hour for 5 miles. What is the bicyclist's average speed? (*Hint:* Average speed equals distance divided by time.) $13.\overline{3}$ mph

Exercises 109 and 110: ***Traffic Flow*** *(Refer to Example 6.) The figure shows a graph of the waiting time T in minutes at a construction zone when cars are arriving randomly at an average rate of x cars per minute.*

(a) *Give the equation of the vertical asymptote.*

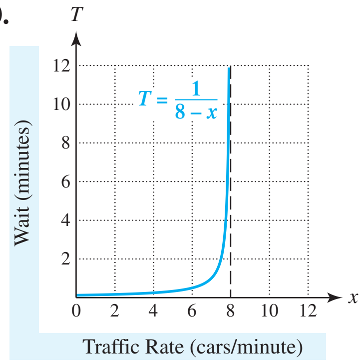
(b) *Explain how the graph relates to traffic flow.*

109.



(a) $x = 5$
(b) As the average arrival rate nears 5 cars/min, a small increase in x increases the waiting time dramatically.

110.



(a) $x = 8$
(b) As the average arrival rate nears 8 cars/min, a small increase in x increases the waiting time dramatically.

WRITING ABOUT MATHEMATICS

111. What is a rational expression? When is a rational expression undefined?

112. Does the rational expression $\frac{5x + 2}{10x + 4}$ equal $\frac{5x}{10x} + \frac{2}{4}$? Explain your answer.

Group Activity Working with Real Data

Directions: Form a group of 2 to 4 people. Select someone to record the group's responses for this activity. All members of the group should work cooperatively to answer the questions. If your instructor asks for your results, each member of the group should be prepared to respond.

Students Per Computer In the early years of personal computers, school districts could not afford to buy a computer for every student. As the price of computers decreased, more and more school districts were able to move toward this goal. The following table lists numbers of students per computer during these early years.

Year	1983	1985	1987	1989
Students/Computer	125	50	32	22

Year	1991	1993	1995	1997
Students/Computer	18	14	10	6

Source: Quality Education Data, Inc.

(a) No; the data are not linear.

(a) Make a scatterplot of the data. Would a straight line model the data accurately? Explain.*

(b) Discuss how well the formula

S models the data quite well.

$$S = \frac{125}{1 + 0.7(x - 1983)}, \quad x \geq 1983$$

models these data, where S represents the students per computer and x represents the year.

(c) In what year does the formula suggest that there were about 17 students per computer? 1992